

Images of the Ammann Beenker Tiling

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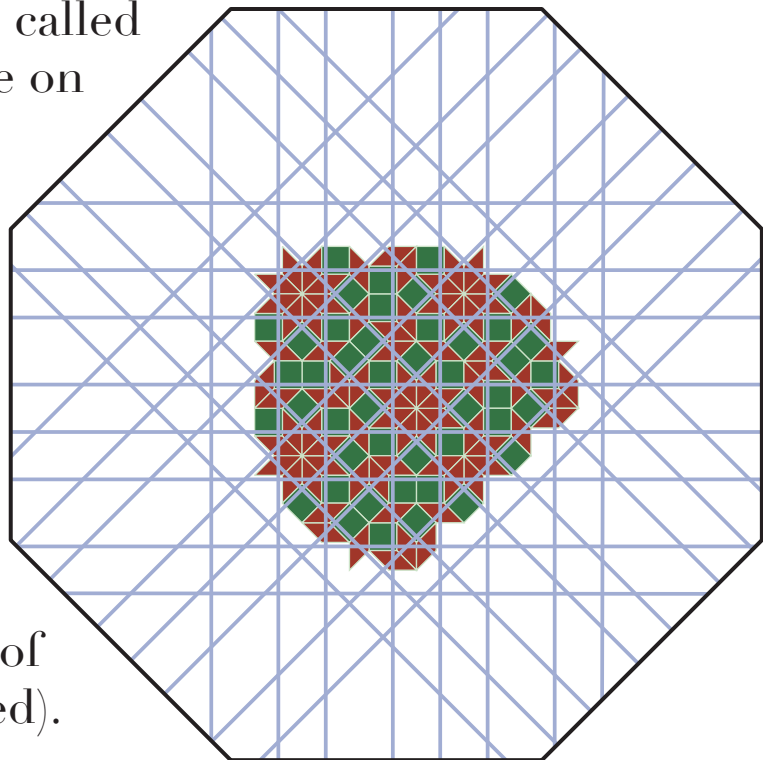
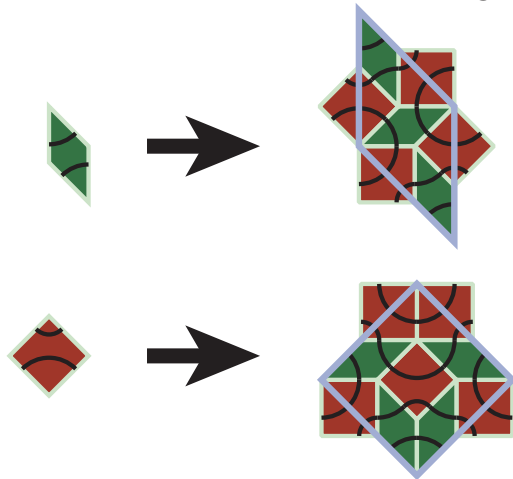
The Ammann-Beenker tiling is the eight-fold sibling of the more famous, five-fold Penrose rhomb tiling. It was discovered independantly by R. Ammann [AGS02] and F. Beenker [Bee82]. Like the Penrose tiling, the Ammann-Beenker can be constructed

by two particular methods.

□ The first method is uses the substitution rule on the left of this sheet. A substitution rule has two phases, the tiles are expanded and then the new tiles are replaced by patches of the original tiles. This can be repeated giving larger patches of tiling and, at the infinite limit, tilings of the whole plane.

□ The second method is to construct the tiling as a planar slice of a four dimensional lattice (in much the same way that a computer draws a line using the pixels of its screen) and then project this to the plane. One consequence of this general method is that tiles with parallel edges lie along lines, called Ammann bars, shown on this page on the right.

□ The two pictures illustrate these two construction methods. The picture on the left shows the hierarchic construction. The picture on the right has tiles on the same line with the same colour for three of the four directions (as every tile lies on two lines, after colouring three of the directions, every tile is coloured).



[AGS92] R Ammann, B Grünbaum and G C Shephard, *Aperiodic Tiles*,

□ □ □ □ *Discrete Comput. Geom.* **8** (1992), 1–25.

□ □ □ □ This paper reports on several of the tilings of Ammann, who was a post-office clerk not a professional mathematician. He had originally found the A-B tiling about 15 years earlier.

[Bee82] □ F P N Beenker, *Algebraic theory of non-periodic tilings of the plane by two simple building*

□ □ □ □ *blocks: a square and a rhombus*, TH-Report 82-WSK04, 1982