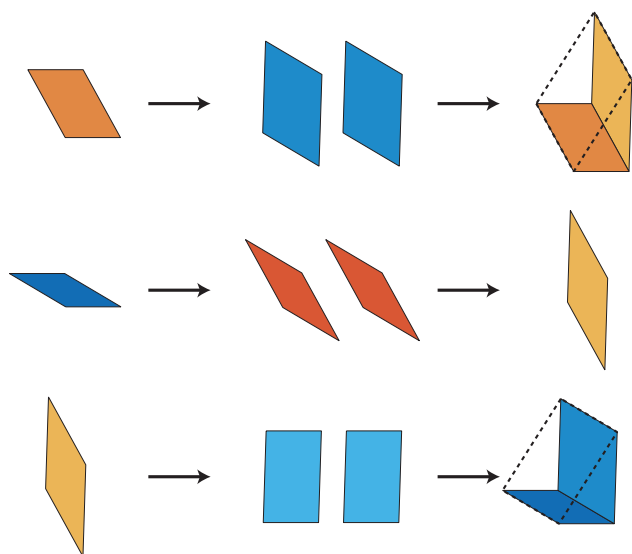


Nautilus and Conch

Edmund Harriss

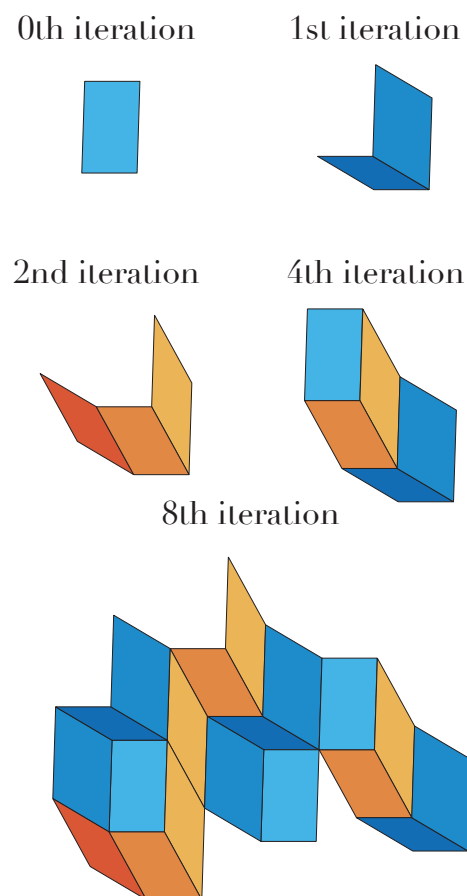
Nautilus and Conch, are two substitution tilings of the plane. A substitution tiling is a tiling produced by a substitution rule, which has two phases. First the tiles are expanded. Then the expanded tiles are replaced by copies of the original tiles. By iterating the substitution rule one may obtain larger patches of the tiling and, in the limit, tilings of the whole plane. Found by EH¹, with P. Arnoux, M. Furukado and S. Ito [AFHI], they were the first non-PV² generalisation of the work of Rauzy, Arnoux and Ito on Rauzy fractals [Fogg02]. In particular, using the two substitution rules, one can explicitly construct the Markov partition of a hyperbolic automorphism of the 4-torus.



the two substitution rules, one can explicitly construct the Markov partition of a hyperbolic automorphism of the 4-torus.

□ The two tilings are locally equivalent to substitution tilings with parallelograms. The fractal bounded tiles of Nautilus can be constructed using this. Begin with the parallelogram substitution rule shown on the left. By iterating on a particular tile we obtain a sequence of shapes. Taking the limit of this sequence for each tile we obtain the tiles of the picture. As shown for a tile on the right. The resulting tiling has the property that it is *Volume Hierarchic*. This means that the image of a tile under the substitution rule is simply a partitioned version of the expanded tile.

□ [AFHI] □ P Arnoux, M Furukado, E Harriss and S Ito, *Algebraic numbers and automorphisms of free groups*, (In preparation).
 [Fogg02] □ Pytheas Fogg, *Substitutions in dynamics, arithmetics and combinatorics*, L N M vol. 1794, Springer-Verlag, Berlin, 2002



¹ At the time EH was a postdoc here in Queen Mary's School of Mathematical Sciences.

² A PV (Pisot-Vijayaraghavan) number is a real, algebraic number whose absolute value is greater than 1 but all of whose algebraic conjugates have absolute values less than 1.