Academic Prejudice and the Spirit of Humbleness

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At the 2008 WKD conference the ivory tower, I did not realise I was in, was rudely shaken. I had not given the cultural divide much thought, following my own path through university and into mathematics. My love of mathematics comes from its power and the way an elegant argument can make the complex simple. My research looks at tilings and patterns, trying to find the simple rules that can generate complex and beautiful images. I also work to try to communicate the beauty and mystery of mathematics in various activites; websites[1], blogs[2], public science exhibitions[3], as well as developing teaching tools.

At the WKD conference I felt a fierceness of rivalry and criticism between different academic disciplines that shocked me. I felt the criticism too often became personal. Much of the rivalry I perceived simply missed the point of what people on the other side were trying to achieve. The fact that it could take place at a conference like the WKD, specifically aimed at communication, scared me. I had gone there expecting to find kindred spirits, like-minded people who shared my own conception of what knowledge embodies; people who would assume that the other participants had something worthwhile to say. Instead, I struggled to suppress frustration at the depths of the differences. To my even greater shock, it gradually dawned on me that other participants might find it just as frustrating to communicate with me.

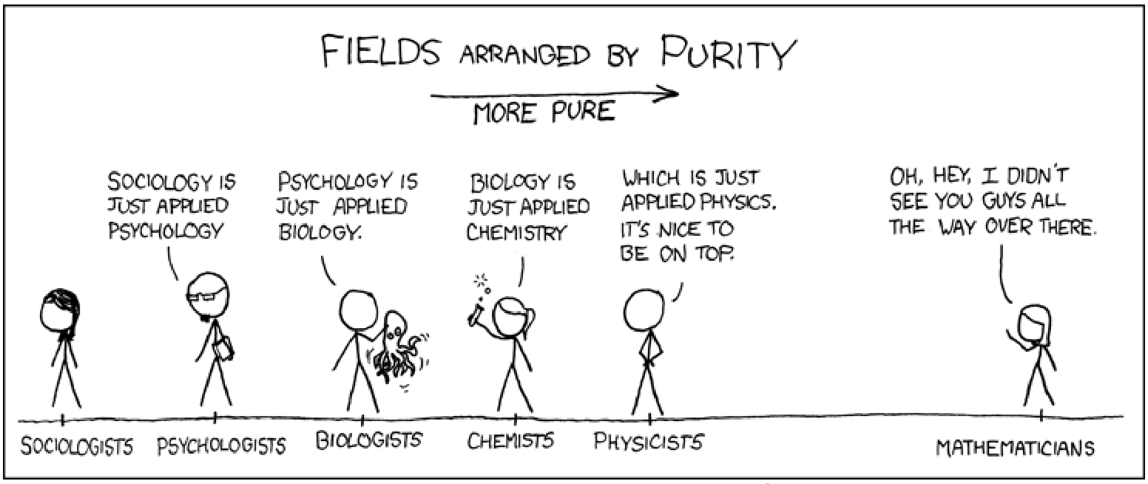
I wanted to respond directly to this. So, confident in the power of my mathematically trained thought, after the conference I started to write this essay. I was quickly brought back to earth. A social scientist with whom I have had several exchanges since WKD said frankly: "I think you could compare it to me writing an essay on geometry without mentioning Euclide, Thalés and so on, and just talking about the way I feel when I look at triangles, saying that there are big and small triangles, etc."

[1] http://www.mathematicians.org.uk/eoh/
[2] http://maxwelldemon.com/
[3] http://www.tilings.org.uk/shapes

He had a point. In many ways I was trying to use the wrong tools for the job. I was taking an immensely complex and subtle issue and trying to deal with it in the same way I would build a mathematical proof. We do need to analyse and understand how science works, yet like many scientists I did not have all the skills necessary for the job. Science needs the input of the human and social sciences not least to bridge the gap with the population at large and move scientific understanding into general knowledge. This is not a one way process however. Just as scientists often have problems communicating their understanding, those who study and do research in the humanities can have problems communicating their understanding of science to people working in the physical sciences. This essay is therefore both a plea to others and an exercise for myself; lets take a step back, be humble and try to understand our own ignorances and respect the knowledge of others. It sounds so obvious, yet it is less simple than one might assume. To this end I present my personal view of science and hope that, at the very least, it gives a data point.

1 Some stereotypes

I define myself as both a mathematician and an artist, so I have always liked to believe that I stand above the great arts versus science debate. However I have to admit that my prejudices and jokes tend to lie more on the science side. Especially as a mathematician I enjoy a cartoon like this:



Purity, ©Randall Munroe xkcd.com/435

However I also see there could be alternatives to this view and hope that the account below, though personal, is reasonably balanced.

The cartoon above illustrates a common cliché of the spectrum of knowledge. On the one side lie the disciplines that most reduce the nature of the world to a language of formulae and equations. Chemistry can then be derived from Physics and Biology from Chemistry. From Biology we can develop Anthropology and Psychology, and thus to Sociology. At the other end of this reductionist representation of the spectrum of human endeavour we might agree to place the Creative Arts. Irrespective of the subjective choice of specific areas that we would place on our personal version of this cartoon, we can probably agree that it resembles a continuum from real life; in every discipline there are people who complain about those in other disciplines in two ways. On one side they say that the results are trivial or that the soul is being removed to reduce everything to an equation[4]. On the other they say the results are not rigorous, or even worse, just empty rhetoric.

At every level, however, there is a remarkable consistency in what people are trying to achieve. They are all trying to understand the world, seeking to find truth, beauty and give meaning both to their own lives and our perception of the world. This is one of the great quests of the human spirit. Whatever we choose to believe in personally, whatever tools we believe can help question or establish our beliefs, we can surely agree at least on one point; Mankind itself is not all-seeing, all-knowing and infinitely-wise and never can be. Even if our neanderthal emotions and stone-age ethics were ever to catch up with our god-like technological capacities, it would be a myth to presume that we could control every parameter that impinges on our individual future or that of our species. Paradoxically, this is not a logical argument against our trying to expand the breadth and depth of mankind's understanding.

2 Complex is not random

Everyone involved in research, even just at a personal level, has experienced that solving problems in most cases opens more questions than it answers. The world may be vastly complex, but it is not random. There are some points of simplicity. The beauty of science has been to find those points that are so simple they can be put into a mathematical language. Similarly the lack of mathematical models and repeatable experiments at the other end of the cartoon's intellectual spectrum just reflects that more precise (and thus simple) tools often fail because the questions are more complex.

So the reciprocal criticisms described above are both valid, but miss the important point. The right end (in the cartoon)[5] is simple, because it has to be. The left end is not rigorous because it cannot be. Let us start with an assumption, the principle of intellectual equality, that the smart people studying all disciplines of knowledge are equally smart. If a subject seems to do more or less, it is firstly because of the abilities and limitations of the tools we have available, not the shortcomings of the practitioners. This is not to say that there are practitioners who do not have shortcomings, but they too are spread over all areas of knowledge. Of course, the way financial resources are distributed in education and research is not free of market forces. If we can agree on this point, it stands to reason that at some level, it must inevitably have a dynamic feedback on what research areas attract

[4]There are even suggestions that engineers are more likely to become terrorists because "characteristics such as greater intolerance of ambiguity, a belief that society can be made to work like clockwork, and dislike of democratic politics which involves compromise, are more common among engineers."Gambetta and Hertog (2009a,b)

[5]I found here that putting so abstract something on a linear scale and talking about it are not as easy as they seem. My initial choice to describe relative positions on this scale (from the cartoon) was to use "above" and "below", but these are hopelessly tinged with value. Even "right" and "left" immediately bring up the language of the political spectrum. Nevertheless I have decided to stick with them and add this disclaimer that they only refer to the cartoon.

the brightest minds. Yet it would be ridiculous to suggest there is any simple correlation between genius and either financial gain or an easy career path.

The tools available to science improve the whole time, but *science* defined as the combination of the experimental method with mathematical models, is not slowly pushing aside human argument and debate. Mathematics creates many of the tools and the models (numbers, differential equations, graphs) that are used by science. We have only a limited number of mathematical tools and have to be able to translate the world into the languages they understand. Mathematics, however tells us more than this. It can reveal the limits of our models and even of human knowledge itself. We should accept that we are stuck with two extremes for our methods of understanding and interpreting the world. In moving forward from present to future there is no single correct path, no predefined recipe for success, even if retrospective analyses allow us to perceive and learn from past mistakes. Each approach to understanding provides insights and the best way forward must surely be to ensure that every possible method of enquiry is fully exploited.

3 Beyond interdisclipinarity

The appeal to a combined approach goes beyond some common notions of interdisciplinary research. In fact an over-simplistic view of interdisclipinary research (often institutionally pushed by insisting departments work together) probably does more damage than good. We must work out how the skills of finding the simple and of truly understanding the complex can be combined. We can never be sure what new avenues of beautiful simplicity remain to be discovered. Yet we must also guard against false assumptions and bad simplification. We need to take better account of how our knowledge can effect what we seek to study. Finally we should consider science itself, how it works and establishes itself. It is not only the mathematician who often seems to forget that, however objective and independent his science, it remains an enterprise conducted by people. Arguably, his science may aspire to purity but he is certainly not a creature of pure logic. As the cartoon cliché reminds us, all acquired knowledge is by definition handed down through human vessels. If you think this is a trivial point, witness how frequently the notions of dialogue between individuals and of dialogue between disciplines of knowledge are freely mixed, in writing and discussion.

This makes for a messy development of knowledge. Let us embrace that, not just as inevitable, but as something we can take advantage of. Specialism should be maintained, yet contacts and personal links should be encouraged as widely as possible. Allowing the possibility for truly wild connections to be made. The true interdisciplinary breakthroughs did not come because two groups of academics were put in a room and told to come up with something. They grew out of individual realisations that two sets of personal skill and knowledge could be productively combined. To achieve this we need humbleness and open minds. We need to accept that even within our expertise there are problems that might be solved, or at least simplified, by a different approach. Similarly we need to accept that when we venture into another's area of expertise they have a good chance of being right if they do not accept our possible solution.

4 From seeing to science

The scientific model gives us a very thorough understanding of the things that we can apply it to. The laws of physics can tell us about the history of the universe itself. They have also revealed what we are made of at unimaginably small sizes. Humankind's most amazing as well as most awful technical achievements, from the creative and connective power of the internet to the destructive power of an atomic bomb, could not have come about without this scientific understanding and simplification. Furthermore it is only retrospectively that it becomes easy to work out where the simple models can be applied. For centuries of cathedral builders in Europe"The marvellous construction of vault and buttress...was the result of a series of ad hoc solutions to practical difficulties." (Bernal, 1971). It would have seemed clear to them that the rules defining the size of a buttress could not be related to those that said how many windows could be in a wall. Many other rules of thumb, intuitions and vernacular knowledge would apply to every part of the building. Yet nearly all these ideas were swept aside by the mechanics of Isaac Newton, whose principles underlie our ability to build structures today that dwarf those early cathedrals. A more recent example of a physics/maths-based insight into an age-old societal problem (conflict) suggests there are remarkable universal similarities between the dynamics of present-day wars and insurgencies in different parts of the world. These modern conflicts seem to show a power law distribution with coefficient around 2.5. It has been suggested this quantitative similarity reveals information about the structure of conflict, which could radically change our worldview of the issue (Johnson, 2006). Thus even in one of the most chaotic and complex of human activities, war, we may be able to catch glimpses of simplicity.

These two examples illustrate a key factor in the progress of science. To model something you first have to be able to see and measure it. Newton's laws could not have been discovered before the telescope was invented and it is no coincidence that their discovery came a relatively short time after. However one step did not follow automatically from the other; Newton's discovery was still the work of genius.

Today we are becoming able to measure and quantify more and more things. The internet itself offers the possibility of accumulating massive data sets that might reveal hidden structure in many chaotic systems. Genomic organisation and DNA sequencing data offers us access to an incredible wealth of information about nature and humanity. One specific example of where it has revolutionised our knowledge of the past is the study of human population flow. Previously this could only be studied by painstaking archeological methods yielding tiny amounts of data. Being able to read the genetic code of the people now living has opened a whole new window on the past (Cavalli-Sforza et al., 1994). The potential for human genius to exploit such tools will depend on how rapidly and fully it can move beyond outmoded and inappropriate concepts of ownership over fields and techniques.

5 New models

Eugene Wigner famously commented on the "Unreasonable effectiveness of mathematics" (Wigner, 1960), that it is amazing where mathematics turns up and the links it reveals. I take a slightly different view. Mathematics provides us with certain models of the world; but it is the power of human imagination that has discovered how these models could be applied to the world. Mathematics provides not just some of our models to understand the world, but every model[6] we have. Luckily the study of mathematics is not static and over time new models are created. It is a tribute to human ingenuity that these models often quickly find unexpected or unpredicted use in modelling the world, a famous example being the use of non-euclidean geometry in the general theory of relativity Gray (1989). In fact sometimes discoveries already made cannot be articulated as such until the mathematical language has been prepared. In 1931 three respected crystallographers published a detailed study that questioned one of the fundamental assumptions of their field (Goldschmidt et al., 1931). Their work was considered mathematically impossible, so it was not until the 1980s that the structures they had considered were recognised. What had happened? In the meantime mathematicians had discovered aperiodic tilings, such as the Penrose tiling. Such patterns are defined by a lack of translational symmetry. They are highly organized structures yet they not periodic (ie there is no patch of tiles that is simply repeated). The strange quasicrystals found by Shectmann in 1984 (Shechtman et al., 1984) could therefore be modelled and thus were able to enter the canon of science (Chapuis, 2003).

6 The limits of science

As time progresses, therefore, better methods of measurement and new mathematical models combine in unpredictable ways with human ingenuity to open up new areas in which science can be used. However the idea that science will slowly grow to take over all of human knowledge is something we should neither hope for nor fear. It is simply not possible. Firstly, though we will no doubt continue to find surprising ways to measure what could not be previously measured (such as DNA) there will always be many things that can never be measured in any realistic way. As well as constructing the models that mimic selected facets of reality, mathematics can study their limits. One problem that turns up again and again is instability or chaos. Once any system gets sufficiently complicated (and that level of complexity is not high at all) it starts to become unpredictable. Think of throwing a stone with a catapult. The force the catapult uses to project the stone is reasonably consistent. Knowing a few parameters beforehand we can do some calculations and generate a theoretical prediction of where the stone will land. This system is not chaotic. If we fire the catapult several times without changing anything the stones will fall quite close to each other and the place predicted by our model. Now consider the weather. We actually have very highly-developed mathematical models of the weather. Yet, unlike our catapult and stone example, if we change the starting data in the model very slightly, the

[6]This is probably a slight overstatement depending on your definition of a model. However anything more than the most basic models have a significant abstract, and thus mathematical component.

predictions can change dramatically. It is important to note that complex systems are not completely random. In many (such as the weather) it is thus possible to make long-term predictions of average, but not detailed, behaviour. Such sensitivity to initial conditions is called chaotic (Grebogi and Yorke, 1997; Stewart, 2002). Not to forget that many systems we try to study in a quantitative manner (for instance, the epigenetic regulation of our chromatin or the fluctuations of financial currency markets) are far more complex than the weather.

Mathematics has shown there are even stronger limits to our ability to interpret the world through a purely rational lense. The mathematicians Kurt Gödel and Alan Turing achieved something that no scientific method had done before (and few have done since). In adding to the body of human knowledge, they managed to demonstrate it has some inherent boundaries. The two concepts that they discovered are closely related. Gödel showed that any axiomatic system that was powerful enough to include arithmetic could neither be complete or proved to be consistent; A system is considered complete if any statement that can be written in the language is either true or false; A system is considered consistent if it does not claim that both a statement and its negation are true, in other words no contradiction exists. Gödel's theory was developed from a rigorous study of statements of the form "This sentence is false.", which are true if false and false if true. Turing's work is closely related, but developed for computer science rather than logic. He showed that there were some questions, (called undecidable), that could not be answered by any computer program. These ideas are very deep, and have been brilliantly expounded by Douglas Hofstadter (Hofstadter, 1979), so I will not try to explain them further here. Essentially they lead to the conclusion that there are limits to rational or computational research. Initially it was hoped by mathematicians that the limitations would only effect the esoteric corners of their subject. Today, however, we know they touch on many areas, for example the work of Greg Chaitin has shown that undecidable problems run to the heart of number theory (Chaitin, 1998).

7 Fooling ourselves

Science then is able to find beautiful simplicities and ways of looking at the world and does it in ways that can surprise us. On the other hand as individuals and as a group, our species is faced with many topics that appear urgent to understand and that science cannot tackle now (even if it might in the future). These questions might be about society, education, war or peace, not to mention love and beauty. People who do research in these complex areas should be ready to accept that so-called pure science might still have something useful to say about what they do. In turn, however, scientists working in the simpler areas should be ready to accept that they have something to learn from those who work every day in the more complex settings. This is where communication can make a real difference, no matter how frustrating it may feel sometimes. In particular it is important to realise when the simplifications we have established are false. As an example, take the issue of homosexuality in animals. For a long time it was claimed that there was no data for the existence of homosexuality other than in humans. Yet one of the problems with collecting the data came from species where the males and females are hard to tell apart. A biologist observing the animals could not sex each individual. They would therefore assume any sex act that took place was heterosexual and, where the gender of one animal was known, ascribe the opposite to the other. This particular false simplification was revealed not by a biologist but by someone who trained as a linguist (Bagemihl, 1999). In the quest for simplicity and understanding it is not that unusual for scientists to fool themselves or be fooled. The magician Harry Houdini made a career out of exposing mediums and spiritualists, many of whom had been endorsed by scientists (Houdini, 1924).

8 Social construcion and logic

This leads to what is perhaps the stickiest point in the clash of intellectual cultures: the debate about whether or not science is socially constructed. I agree with the viewpoint that it is. An alternative version of the debate is about whether science is merely a belief system. Again I agree that at heart it is, but we have to approach both these ideas with subtlety. The scientific method is an effective social construction and provides a very powerful way of producing agreement. This is what allows the common body of knowledge to grow. The strength of science is perhaps better seen not in that it can produce objective truth, but that it can at least identify falsehood. In my experience at least, most scientists spend a long time analysing their methods with care and skepticism. They give deep thought to how an experiment can answer a question and are constantly checking for flaws in their arguments or alternative interpretations of their results. This procedure applies to the discipline of logic itself. In some versions of relativism or social construction our universally accepted system of logic is introduced as just one of many possible, and equally valid, systems. Paul Boghossian has taken this idea on, arguing that it does not have philosophical backing (Boghossian, 2006). I would like to take a different approach. I would love to hear more speculation about alternative logical systems. In fact mathematicians have thought long and hard about this. At the beginning of the twentieth century it was one of the largest topics of mathematical study. A particular system that was studied in depth was Intuitionism proposed by Brouwer as an alternative to the Formalism of Hilbert (Mancosu, 1998). These were essentially the same systems but the first rejected the idea of proof by contradiction (if I can prove that something is false implies it is true then the statement must be true). It led to proofs in many areas of mathematics being re-derived by alternative methods (for example non-standard analysis (Robinson, 1996)). This debate has essentially concluded, though it was not ended by the discovery of some perfect system but by Gödel's arguments that such a system cannot exist.

As a result even mathematics requires some elements of belief; statements that must be assumed rather than proved. Such statements are called axioms and much work has been done on the foundations of mathematics looking at what the simplest collection of statements might be. One of the most powerful methods is to build mathematics from set theory. This is based on the basic idea of a set (naively just a collection of things). The most commonly used axioms for set theory are the ZF axioms (named after their creators Ernst Zermelo and Abraham Fraenkel)Jech (2003). These axioms are incredibly simple. For example one (of just 10) states that two sets are the same if they contain the same things. Another axiom says that given two sets the collection of things in one or both of them is itself a set (the union). Personally I find these elements are statements that would be as hard to refute as statements about my own existence, but I accept that they are still unproven beliefs. Mathematicians have not come up with any alternative system with the same power as this classical one, but with different conclusions. This is evidence, though not proof, that no such powerful alternative system can exist.

I believe therefore that it is not useful to apply the concept of social construction to logical systems. Social construction is, however, a powerful tool to help understand how science itself works. Within a restricted area of research one can ask how much the acquisition of understanding affects how the system you study works. Personally I would not give much time to a claim that the discovery of Newton's laws changed the way the universe worked, but we have no way to test this. In Biology, however, it is regarded as usual good practice to make many controls designed to minimise (but not eliminate), the risk that an experimenter's actions have inadvertently influenced the outcome of his experiment or his interpretation of data. A wrong decision can lead to bad data, as in the the study of homosexuality in animals mentioned above. If the interpretation of their research is called in to question, many researchers react emotionally as if their personal integrity had been doubted. However it can be illuminating to examine how much supposed facts evolve over time. One comprehensive illustration of this point can be found by comparing modern text books on general human physiology and medicine with some of the reputable texts from one hundred years ago Porter (1997).

At the other extreme of the cartoon's scale, there are situations where there is universal agreement that the objects of study are socially constructed; the classic example being money. In the science of economics, there are simple rules to be discovered. The relationship between supply and demand and prices for example can be modelled. In this case the understanding is pushed back into the system. People hoard goods in an attempt to bring up the price and so on. Economic understanding is overtly used within the system that it models so understanding the rules creates new, more complex rules. In such circumstances we should hold a subtle balance. On the one hand we should not reject the power of the scientific method to provide simplification. On the other we should not reject ideas because they are not part of the model. It is in subjects such as this that we need the most collaboration between styles of thinking. Yet, at least to my outsider's perspective, it appears as the area where the most disagreement between academic cultures takes place. As a result, whatever precious understanding we can achieve by any method is often widely ignored and the forces of popular and financial opportunism have a free hand Weinstein (2009).

9 The Politics of science

Now consider the realm of natural science itself (or equally the humanities). These are immensely complicated societies that are able to generate new knowledge. Sometimes the process seems to come about more by accident than design, though we may need to believe that we plan it. Anyone who has worked in a science department at a university is conscious of an element of internal politics that goes on not to mention the fashions and mini-trends of funding decisions and the biases introduced by overfunding of certain viewpoints. To illustrate this with a historical example, let me commit heresy for a British mathematician; Liebnitz did better than Newton. We shall leave aside the controversy of who actually came up with the calculus. Liebnitz came up with a far superior notation, the $\frac{dy}{dx}$ that we use today. Newton just put a dot over the function for his fluxions. The political rivalry, however, meant that British mathematicians for years afterwards continued to use the less flexible Newtonian notation. It can be argued that it is not until the work of G H Hardy at the start of the twentieth century that British mathematics caught up to the rest of Europe (Wiener, 1949). This sort of issue will get more problematic and complex as the number of people involved in science continues to grow. There is a wealth of excellent literature discussing the nature of science and this merits much more consideration by scientists than it has received. For example Bruno Latour has analysed the ways in which scientific work becomes fact (Latour, 1987). He considers how papers and scientists draw upon the work of the past to position and defend their work and how this is in turn reinterpreted by later papers. Eventually scientific results become black boxes that can only be opened with effort. An example is the first discovery of quasicrystals discussed earlier, a paper that was initially forgotten, yet has now been given a new lease of life as an example of a scientific chance missed. Another insightful analysis of the nature of science is that of David Turnball, who has proposed that our modern system of knowledge is not so different from ancient ones and that "modernity's drive for order concealed its messy contingent, unplanned and arational character."(Turnbull, 2000).

So let me finish by reiterating the plea of Julia Higgins Higgins (2008) let us all be aware of the failings of our own subjects and adhere to the principle of intellectual equality. Let us come to acknowledged experts in other fields with an initial respect; listen to what they have to say about our subject, tell them what we think of theirs and trust that they will listen and not just reject without thought. Finally remember that communication between different levels of the original cartoon's specturm is *hard*. When we fail it is much more often because of the complexity involved, than because of any stupidity or ignorance.

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