

M3/4PA45 Tilings and Patterns

Problems 7

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Problem 1

Find the window for the first vertex of the patch $abaab$ (where a is horizontal and b vertical) in the Fibonacci tiling.

Solution

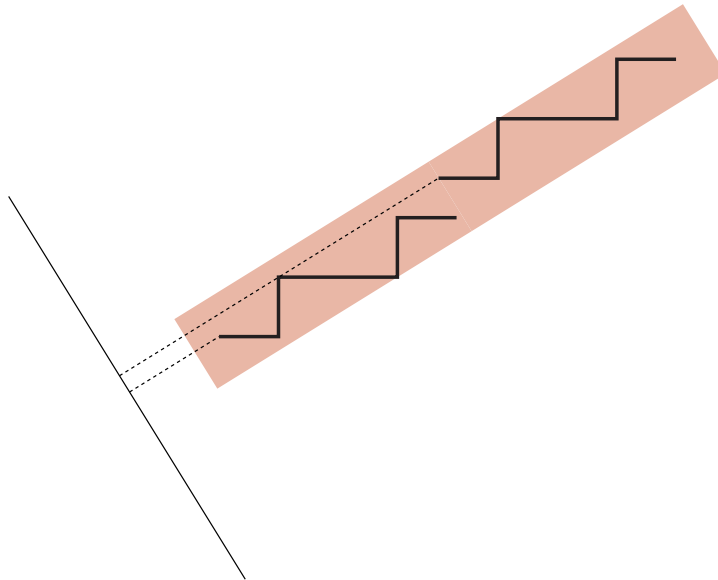


Figure 1: The window for the patch $abaab$.

The window for the patch $abaab$ is given by all translation of the section of the associated staircase in W so that it lies in $V + H$. This is shown in Figure ???. To find this by hand calculate the projections of the vectors $(0, 1)$ and $(1, 0)$, then find the points that give the highest and lowest values in W . In this case these are the points $(3, 1)$ (lowest) and $(1, 1)$ (highest). The boundaries of the window are therefore $t - \Pi_W(3, 1)$ and $t + \Pi_W(-1, 1) - \Pi_W(1, 1)$, where t and $t + \Pi_W(-1, 1)$ are the boundaries of the full window.

Solution

The substitution rule $a \rightarrow abaa, b \rightarrow abaaaba$ is invertible with inverse $a \rightarrow a^{-1}ba^{-1}a^{-1}ba^{-1}a^{-1}b, b \rightarrow b^{-1}aa$, and has incidence matrix $\begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$. As it is invertible it generates canonical projection tilings by Lemma 11.14 in the notes. The incidence matrix gives the projection and therefore they are the canonical projection tilings defined by the desired matrix.

Problem 2

Find a substitution rule that generates the CPT defined by the matrix: $\begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$.

Solution

The substitution rule $a \rightarrow abaa, b \rightarrow abaaaba$ is invertible with inverse $a \rightarrow a^{-1}ba^{-1}a^{-1}ba^{-1}a^{-1}b$, $b \rightarrow b^{-1}aa$, and has incidence matrix $\begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix}$. As it is invertible it generates canonical projection tilings by Lemma 11.14 in the notes. The incidence matrix gives the projection and therefore they are the canonical projection tilings defined by the desired matrix.

Other work

Study the proofs that the CPT for given V and W for an LI class, and that the CPT tilings defined by an expansion matrix have substitution rules.